Consider a light wave propagating in the z direction. For simplicity let the light be linearly polarized with electric field vector $\mathbf{E}$ pointing in the x direction.

$$\mathbf{E}(t,x,y,z)=\begin{pmatrix}A\sin(kz-\omega t)\\0\\0\end{pmatrix}$$

The symbol ω is angular frequency. The symbol k is the wave number $k=\omega/c$. The photon wave function corresponding to $\mathbf{E}$ is

$$\psi=\exp(inkz-in\omega t)$$

where n is the number of photons. The electric field operator $\hat{\mathscr{E}}$ is defined as

$$\begin{aligned}\hat{\mathscr{E}}&=iC(\hat{a}-\hat{a}^\dagger)\\&=iC\sqrt{\hat{n}}\exp(-ikz+i\omega t)-iC\sqrt{\hat{n}+1}\exp(ikz-i\omega t)\end{aligned}$$

The hatted symbols are operators that operate on a wave function. For example $\sqrt{\hat{n}}\psi=\sqrt{n}\psi$ where the hatless n is the number of photons in ψ. The symbol C is a conversion constant defined as

$$C=\sqrt{\frac{\hbar\omega}{2\varepsilon_0}}$$

The electric field operator $\hat{\mathscr{E}}$ has an eigenvalue $\mathscr{E}$ such that $\hat{\mathscr{E}}\psi=\mathscr{E}\psi$. The eigenvalue $\mathscr{E}$ is the observed electric field.

$$\begin{aligned}\mathscr{E}&=\left(\hat{\mathscr{E}}\psi\right)\psi^{-1}\\&=iC\sqrt{n}\exp(-ikz+i\omega t)-iC\sqrt{n+1}\exp(ikz-i\omega t)\\&=C\left(\sqrt{n+1}+\sqrt{n}\right)\sin(kz-\omega t)-iC\left(\sqrt{n+1}-\sqrt{n}\right)\cos(kz-\omega t)\end{aligned}$$

For large n the cosine term in $\mathscr{E}$ vanishes and we have the following approximation.

$$\mathscr{E}=2C\sqrt{n}\sin(kz-\omega t)$$

Identifying $\mathscr{E}$ as the first component of $\mathbf{E}$ we have $A\sin(kz-\omega t)=\mathscr{E}$. Hence the amplitude A is proportional to the square root of the number of photons.

$$A=2C\sqrt{n}$$

The unit of electric field strength is volt meter^{-1}. The following script calculates C for converting number of photons$^{1/2}$ to volt meter^{-1}. Yellow light with wavelength $\lambda=600$ nanometers is used for angular frequency ω. The result is

$$C=\sqrt{\frac{\hbar\omega}{2V\varepsilon_0}}=1.367\times10^{-4}\,\text{volt}\,\text{meter}^{-1}$$

The symbol V is a one cubic meter unit volume. The script also converts volt meter^{-1} to base units.

$$1\,\text{volt}\,\text{meter}^{-1}=1\,\text{kilogram}\,\text{meter}\,\text{ampere}^{-1}\,\text{second}^{-3}$$

```
-- www.eigenmath.org/quantum-electric-field.txt
-- derived units
joule = kilogram * meter^2 * second^(-2)
volt = kilogram * meter^2 * second^(-3) * ampere^(-1)
farad = kilogram^(-1) * meter^(-2) * second^4 * ampere^2
-- physical values
hbar = 1.054572 * 10^(-34) * joule * second
epsilon0 = 8.854188 * 10^(-12) * farad * meter^(-1)
c = 299792458 * meter * second^(-1)
lambda = 600 * 10^(-9) * meter
omega = 2 * float(pi) * c / lambda
-- conversion constant
C = sqrt(0.5 * hbar * omega * epsilon0^(-1) * meter^(-3))
C
volt * meter^(-1)
```