Consider the following eigenstates of a hypothetical quantum system.

$$
\begin{aligned}
|00\rangle &= (1\ 0\ 0\ 0)^\dagger && \text{no fermions} \\
|10\rangle &= (0\ 1\ 0\ 0)^\dagger && \text{one fermion in state } \phi_1 \\
|01\rangle &= (0\ 0\ 1\ 0)^\dagger && \text{one fermion in state } \phi_2 \\
|11\rangle &= (0\ 0\ 0\ 1)^\dagger && \text{two fermions, one in state } \phi_1\text{, one in state } \phi_2
\end{aligned}
$$

Let fermion states ϕ_n be modeled by a one dimensional box of length L.

$$\phi_n(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right)$$

Creation and annihilation operators are formed from outer products of state vectors. Sign changes make the operators antisymmetric.

$$
\begin{aligned}
\hat{b}_1^\dagger &= |10\rangle\langle 00| - |11\rangle\langle 01| && \text{Create one fermion in state } \phi_1 \\
\hat{b}_1 &= |00\rangle\langle 10| - |01\rangle\langle 11| && \text{Annihilate one fermion in state } \phi_1 \\
\hat{b}_2^\dagger &= |01\rangle\langle 00| + |11\rangle\langle 10| && \text{Create one fermion in state } \phi_2 \\
\hat{b}_2 &= |00\rangle\langle 01| + |10\rangle\langle 11| && \text{Annihilate one fermion in state } \phi_2
\end{aligned}
$$

Let $\hat{r}$ be the position operator

$$\hat{r} = \sum_{n,m} r_{nm} \hat{b}_n^\dagger \hat{b}_m$$

where

$$r_{nm} = \int_0^L \phi_n^*(x) x \phi_m(x)\, dx$$

Note that for a one dimensional box

$$r_{nn} = \langle x\rangle = \tfrac{1}{2}L$$

Verify that

$$
\begin{aligned}
\langle 10|\hat{r}|10\rangle &= r_{11} \\
\langle 10|\hat{r}|01\rangle &= r_{12} \\
\langle 01|\hat{r}|10\rangle &= r_{21} \\
\langle 01|\hat{r}|01\rangle &= r_{22}
\end{aligned}
$$