PLANE WAVE

Consider a light wave traveling in the z direction with electric vector $\mathbf{E}$ and magnetic vector $\mathbf{B}$.

$$\mathbf{E}(z,t) = \begin{pmatrix} A\sin(kz - \omega t + \phi) \\ 0 \\ 0 \end{pmatrix} \qquad \mathbf{B}(z,t) = \begin{pmatrix} 0 \\ A\sin(kz - \omega t + \phi) \\ 0 \end{pmatrix}$$

The symbol A is the amplitude and ϕ is a phase shift. The angular frequency ω and wave number k are related to frequency f as follows.

$$\omega = 2\pi f$$
$$k = \omega / c$$

For a given t, the vectors $\mathbf{E}$ and $\mathbf{B}$ are constant throughout a plane perpendicular to the z axis. Hence the name plane wave. The electric and magnetic vectors can be derived from a vector potential $\mathbf{A}$. For example, define $\mathbf{A}$ as

$$\mathbf{A} = \begin{pmatrix} -A_x\,\omega^{-1}\cos(kz - \omega t + \phi) \\ 0 \\ 0 \end{pmatrix}$$

The electric and magnetic vectors are computed from $\mathbf{A}$ as

$$\mathbf{E} = -\frac{\partial}{\partial t}\mathbf{A} = \begin{pmatrix} A_x\sin(kz - \omega t + \phi) \\ 0 \\ 0 \end{pmatrix} \qquad \mathbf{B} = c\nabla \times \mathbf{A} = \begin{pmatrix} 0 \\ A_x\sin(kz - \omega t + \phi) \\ 0 \end{pmatrix}$$

The following script computes $\mathbf{E}$ and $\mathbf{B}$ from $\mathbf{A}$ and shows that $\mathbf{E}$ and $\mathbf{B}$ are solutions to the free-field Maxwell equations

$$\nabla \cdot \mathbf{E} = 0 \qquad \nabla \times \mathbf{E} + c^{-1}\frac{\partial}{\partial t}\mathbf{B} = 0$$

$$\nabla \cdot \mathbf{B} = 0 \qquad \nabla \times \mathbf{B} - c^{-1}\frac{\partial}{\partial t}\mathbf{E} = 0$$

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-- www.eigenmath.org/plane-wave.txt
k = omega/c
A = (-Ax*cos(k*z-omega*t+phi)/omega, 0, 0)
-- compute E and B
E = -d(A,t)
B = c * curl(A)
-- show E and B
E
B
-- check maxwell equations
div(E)
div(B)
curl(E) + d(B,t)/c
curl(B) - d(E,t)/c
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