Prove the following Gordon decomposition by direct calculation. Momentum vectors p_1 and p_2 have the same rest mass m. Each of the spins s_1 and s_2 can be either up or down.

$$\bar{u}(p_2,s_2)\gamma^\mu u(p_1,s_1) = \bar{u}(p_2,s_2)\left[\frac{(p_2+p_1)^\mu}{2m} + i\sigma^{\mu\nu}\frac{(p_2-p_1)_\nu}{2m}\right]u(p_1,s_1)$$

The following vectors and spinors are used. Spinors u_{11} and u_{21} are spin up, u_{12} and u_{22} are spin down.

$$p_1 = \begin{pmatrix} E_1 \\ p_{1x} \\ p_{1y} \\ p_{1z} \end{pmatrix} \quad u_{11} = \begin{pmatrix} E_1+m \\ 0 \\ p_{1z} \\ p_{1x}+ip_{1y} \end{pmatrix} \quad u_{12} = \begin{pmatrix} 0 \\ E_1+m \\ p_{1x}-ip_{1y} \\ -p_{1z} \end{pmatrix} \quad E_1 = \sqrt{p_{1x}^2+p_{1y}^2+p_{1z}^2+m^2}$$

$$p_2 = \begin{pmatrix} E_2 \\ p_{2x} \\ p_{2y} \\ p_{2z} \end{pmatrix} \quad u_{21} = \begin{pmatrix} E_2+m \\ 0 \\ p_{2z} \\ p_{2x}+ip_{2y} \end{pmatrix} \quad u_{22} = \begin{pmatrix} 0 \\ E_2+m \\ p_{2x}-ip_{2y} \\ -p_{2z} \end{pmatrix} \quad E_2 = \sqrt{p_{2x}^2+p_{2y}^2+p_{2z}^2+m^2}$$

Tensor $\sigma^{\mu\nu}$ is defined as

$$\sigma^{\mu\nu} = \frac{i}{2}\left[\gamma^\mu,\gamma^\nu\right] = \frac{i}{2}\left(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu\right)$$

In component notation we have

$$\sigma^{\mu\nu\alpha}{}_\beta = \frac{i}{2}\left(\gamma^{\mu\alpha}{}_\rho\gamma^{\nu\rho}{}_\beta - \gamma^{\nu\alpha}{}_\rho\gamma^{\mu\rho}{}_\beta\right)$$

Let $T^{\mu\nu} = \gamma^\mu\gamma^\nu$. Transpose the first two indices of $\gamma^{\nu\rho}{}_\beta$ to form a dot product.

$$T^{\mu\nu\alpha}{}_\beta = \gamma^{\mu\alpha}{}_\rho\gamma^{\rho\nu}{}_\beta$$

Convert to code. The transpose on the second and third indices interchanges α and ν.

$T^{\mu\nu\alpha}{}_\beta =$ `transpose(dot(gamma,transpose(gamma)),2,3)`

Hence

$\sigma^{\mu\nu} =$ `i/2 * (T - transpose(T))`

where `T` $= T^{\mu\nu\alpha}{}_\beta$. The inner product $\sigma^{\mu\nu}(p_2-p_1)_\nu$ is computed with the dot arguments in a specific order.

$\sigma^{\mu\nu}(p_2-p_1)_\nu =$ `dot(gmunu,p2-p1,sigmamunu[mu])`

Multiplication by the metric tensor lowers the index of p_2-p_1. Tensor $\sigma^{\mu\nu}$ is placed on the right-hand side of the dot product so that index ν is correctly oriented. The complete index for $\sigma^{\mu\nu}$ is $\sigma^{\mu\nu\alpha}{}_\beta$. The μ is removed by indexing leaving $\sigma^{\nu\alpha}{}_\beta$. The dot product sums over ν in $(p_2-p_1)_\nu\sigma^{\nu\alpha}{}_\beta$.

```
-- www.eigenmath.org/gordon.txt
E1 = sqrt(p1x^2 + p1y^2 + p1z^2 + m^2)
E2 = sqrt(p2x^2 + p2y^2 + p2z^2 + m^2)
p1 = (E1,p1x,p1y,p1z)
p2 = (E2,p2x,p2y,p2z)
u11 = (E1+m,0,p1z,p1x+i*p1y)
u12 = (0,E1+m,p1x-i*p1y,-p1z)
u21 = (E2+m,0,p2z,p2x+i*p2y)
u22 = (0,E2+m,p2x-i*p2y,-p2z)
u1 = (u11,u12)
u2 = (u21,u22)
I = ((1,0,0,0),(0,1,0,0),(0,0,1,0),(0,0,0,1))
gmunu = ((1,0,0,0),(0,-1,0,0),(0,0,-1,0),(0,0,0,-1))
gamma0 = ((1,0,0,0),(0,1,0,0),(0,0,-1,0),(0,0,0,-1))
gamma1 = ((0,0,0,1),(0,0,1,0),(0,-1,0,0),(-1,0,0,0))
gamma2 = ((0,0,0,-i),(0,0,i,0),(0,i,0,0),(-i,0,0,0))
gamma3 = ((0,0,1,0),(0,0,0,-1),(-1,0,0,0),(0,1,0,0))
gamma = (gamma0,gamma1,gamma2,gamma3)
T = transpose(dot(gamma,transpose(gamma)),2,3)
sigmamunu = i/2 * (T - transpose(T))
for(s1,1,2,for(s2,1,2,for(mu,1,4,
  T = 1/(2*m) * ((p2+p1)[mu]*I + i*dot(gmunu,p2-p1,sigmamunu[mu])),
  A = dot(conj(u2[s2]),gamma0,T,u1[s1]),
  B = dot(conj(u2[s2]),gamma0,gamma[mu],u1[s1]),
  print(A-B)
)))
```