This demo shows how to obtain a classical plane wave from a photon wave function. Let there be an electromagnetic wave traveling in the z direction with the following electric field vector.

$$\mathbf{E} = \begin{pmatrix} A_x \sin(kz - \omega t) \\ 0 \\ 0 \end{pmatrix}$$

The corresponding photon wave function is

$$\psi_n(z,t) = \exp(in(kz - \omega t))$$

where n is the number of photons. The electric field operator is

$$\hat{E}_x = iC(a - a^\dagger)$$

where C is a conversion constant. Apply the electric field operator to the wave function ψ_n to obtain

$$\begin{aligned} \hat{E}_x \psi_n &= iC(a\psi_n - a^\dagger \psi_n) \\ &= iC\left[\sqrt{n}\psi_{n-1} - \sqrt{n+1}\psi_{n+1}\right] \end{aligned}$$

Recall that the observed electric field E_x is an eigenvalue of $\hat{E}_x$, that is

$$\hat{E}_x \psi_n = E_x \psi_n$$

Divide $\hat{E}_x \psi_n$ by ψ_n to obtain the observed electric field E_x.

$$\begin{aligned} E_x &= \frac{\hat{E}_x \psi_n}{\psi_n} \\ &= iC\left[\frac{\sqrt{n}\psi_{n-1} - \sqrt{n+1}\psi_{n+1}}{\psi_n}\right] \\ &= iC\left[\frac{\sqrt{n}\exp(i(n-1)(kz-\omega t)) - \sqrt{n+1}\exp(i(n+1)(kz-\omega t))}{\exp(in(kz-\omega t))}\right] \end{aligned}$$

The exponentials in the numerator and denominator cancel, hence

$$E_x = iC\Big[\sqrt{n}\exp(-i(kz-\omega t)) - \sqrt{n+1}\exp(i(kz-\omega t))\Big]$$

Note that $\sqrt{n} \approx \sqrt{n+1}$ for large n. For the large n approximation we have

$$E_x = iC\sqrt{n}\Big[\exp(-i(kz-\omega t)) - \exp(i(kz-\omega t))\Big]$$

Recall that $2\sin(\alpha) = i[\exp(-i\alpha) - \exp(i\alpha)]$. Hence

$$E_x = 2C\sqrt{n}\sin(kz - \omega t)$$

which is equivalent to the x coordinate of $\mathbf{E}$ with $A_x = 2C\sqrt{n}$. Note that the amplitude of the electric field is proportional to the square root of the number of photons.

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-- www.eigenmath.org/from-photons-to-maxwell.txt
psi(n) = exp(i * n * (k*x - omega*t))
psi1 = psi(1)
-- define operators
a(psi) = conj(psi1) * sqrt(i / omega * psi * d(psi,t))
adag(psi) = psi1 * sqrt(i / omega * psi * d(psi,t) + psi^2)
Ehat(psi) = i * C * (a(psi) - adag(psi))
-- compute E for 1 million photons
N = 1000000
E = Ehat(psi(N)) / psi(N)
E = subst(-i*C*sqrt(N)*psi1, -i*C*sqrt(N+1)*psi1, E) -- large n approx
-- show E
E
-- compare
E - 2 * C * sqrt(N) * expsin(k*x - omega*t)
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