Electron spin state s is a unit vector in $\mathbb{C}^2$.

$$s = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$$

$$s^\dagger s = \bar{c}_1 c_1 + \bar{c}_2 c_2 = 1$$

These are the spin operators.

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

The expectation value of a spin operator is a projection of s onto Euclidean space.

$$\langle x \rangle = s^\dagger \sigma_x s \qquad \langle y \rangle = s^\dagger \sigma_y s \qquad \langle z \rangle = s^\dagger \sigma_z s$$

Let

$$\sigma = \begin{pmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \end{pmatrix}$$

Then the spin polarization vector P is

$$P = \begin{pmatrix} \langle x \rangle \\ \langle y \rangle \\ \langle z \rangle \end{pmatrix} = s^\dagger \sigma s$$

In component notation $\sigma = \sigma^{\alpha\beta}{}_\gamma$ and

$$P^\alpha = \bar{s}_\beta \sigma^{\alpha\beta}{}_\gamma s^\gamma$$

In Eigenmath a transpose is needed to swap α and β so that summed-over indices are adjacent.

$$P^\alpha = \bar{s}_\beta \sigma^{\beta\alpha}{}_\gamma s^\gamma$$

Hence the code is

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P = dot(conj(s),transpose(sigma),s)
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Let θ and ϕ be polar and azimuth angles such that

$$P = \begin{pmatrix} \sin\theta\cos\phi \\ \sin\theta\sin\phi \\ \cos\theta \end{pmatrix}$$

The corresponding spin state is

$$s = \begin{pmatrix} \cos(\theta/2) \\ \sin(\theta/2)\exp(i\phi) \end{pmatrix}$$