This is the Dirac equation with $c = 1$ and $\hbar = 1$.

$$i\gamma^0 \frac{\partial}{\partial t}\psi + i\gamma^1 \frac{\partial}{\partial x}\psi + i\gamma^2 \frac{\partial}{\partial y}\psi + i\gamma^3 \frac{\partial}{\partial z}\psi - m\psi = 0$$

The following gamma matrices are the "Dirac representation."

$$\gamma^0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \quad \gamma^1 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix} \quad \gamma^2 = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix} \quad \gamma^3 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

The wave function ψ has angular frequency ω equal to the energy of the particle.

$$\omega = \sqrt{k_x^2 + k_y^2 + k_z^2 + m^2}$$

There are four positive frequency solutions that are linearly independent.

$$\psi_1 = \begin{pmatrix} \omega + m \\ 0 \\ k_z \\ k_x + ik_y \end{pmatrix} \exp[i(k_x x + k_y y + k_z z - \omega t)] \quad \psi_2 = \begin{pmatrix} 0 \\ \omega + m \\ k_x - ik_y \\ -k_z \end{pmatrix} \exp[i(k_x x + k_y y + k_z z - \omega t)]$$

$$\psi_3 = \begin{pmatrix} k_z \\ k_x + ik_y \\ \omega - m \\ 0 \end{pmatrix} \exp[i(k_x x + k_y y + k_z z - \omega t)] \quad \psi_4 = \begin{pmatrix} k_x - ik_y \\ -k_z \\ 0 \\ \omega - m \end{pmatrix} \exp[i(k_x x + k_y y + k_z z - \omega t)]$$

There are four negative frequency solutions that are linearly independent. The negative frequency solutions flip the sign of m.

$$\psi_5 = \begin{pmatrix} \omega - m \\ 0 \\ k_z \\ k_x + ik_y \end{pmatrix} \exp[-i(k_x x + k_y y + k_z z - \omega t)] \quad \psi_6 = \begin{pmatrix} 0 \\ \omega - m \\ k_x - ik_y \\ -k_z \end{pmatrix} \exp[-i(k_x x + k_y y + k_z z - \omega t)]$$

$$\psi_7 = \begin{pmatrix} k_z \\ k_x + ik_y \\ \omega + m \\ 0 \end{pmatrix} \exp[-i(k_x x + k_y y + k_z z - \omega t)] \quad \psi_8 = \begin{pmatrix} k_x - ik_y \\ -k_z \\ 0 \\ \omega + m \end{pmatrix} \exp[-i(k_x x + k_y y + k_z z - \omega t)]$$

This script verifies the solutions.

```
-- www.eigenmath.org/dirac.txt
gamma0 = ((1,0,0,0),(0,1,0,0),(0,0,-1,0),(0,0,0,-1))
gamma1 = ((0,0,0,1),(0,0,1,0),(0,-1,0,0),(-1,0,0,0))
gamma2 = ((0,0,0,-i),(0,0,i,0),(0,i,0,0),(-i,0,0,0))
gamma3 = ((0,0,1,0),(0,0,0,-1),(-1,0,0,0),(0,1,0,0))
omega = sqrt(kx^2 + ky^2 + kz^2 + m^2)
psi1 = (omega+m, 0, kz, kx+i*ky) * exp(i*(kx*x + ky*y + kz*z - omega*t))
psi2 = (0, omega+m, kx-i*ky, -kz) * exp(i*(kx*x + ky*y + kz*z - omega*t))
psi3 = (kz, kx+i*ky, omega-m, 0) * exp(i*(kx*x + ky*y + kz*z - omega*t))
psi4 = (kx-i*ky, -kz, 0, omega-m) * exp(i*(kx*x + ky*y + kz*z - omega*t))
psi5 = (omega-m, 0, kz, kx+i*ky) * exp(-i*(kx*x + ky*y + kz*z - omega*t))
psi6 = (0, omega-m, kx-i*ky, -kz) * exp(-i*(kx*x + ky*y + kz*z - omega*t))
psi7 = (kz, kx+i*ky, omega+m, 0) * exp(-i*(kx*x + ky*y + kz*z - omega*t))
psi8 = (kx-i*ky, -kz, 0, omega+m) * exp(-i*(kx*x + ky*y + kz*z - omega*t))
D(psi) = i*dot(gamma0,d(psi,t)) +
         i*dot(gamma1,d(psi,x)) +
         i*dot(gamma2,d(psi,y)) +
         i*dot(gamma3,d(psi,z)) - m*psi
D(psi1)
D(psi2)
D(psi3)
D(psi4)
D(psi5)
D(psi6)
D(psi7)
D(psi8)
```